A general framework for topological phases with space-time symmetries

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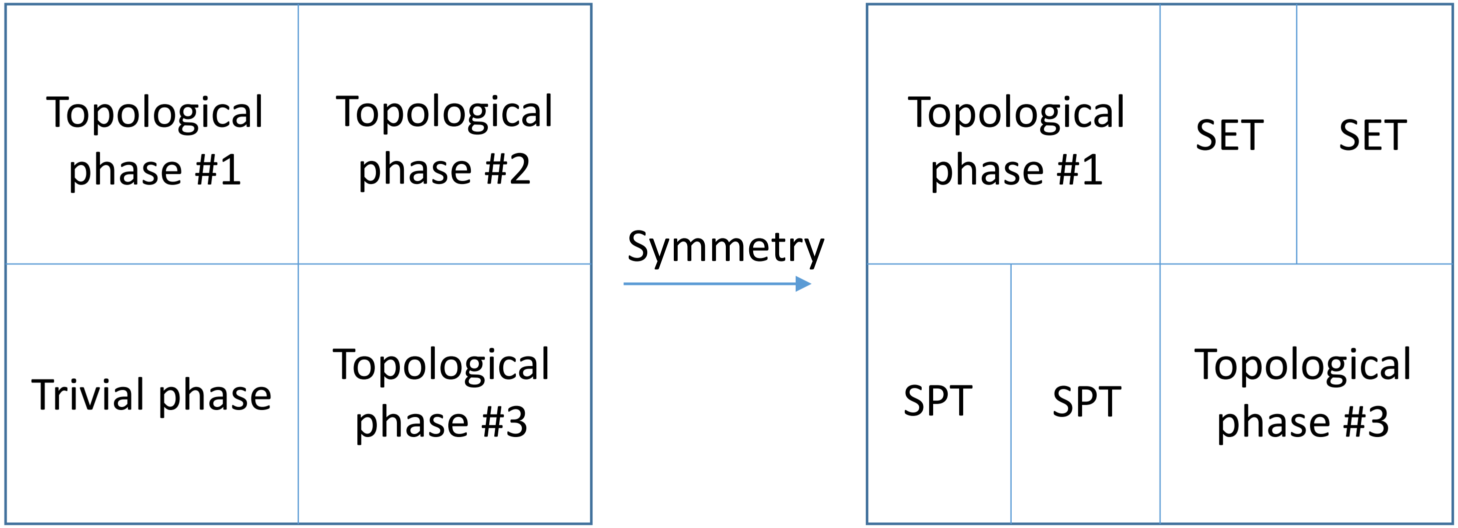
I. Introduction

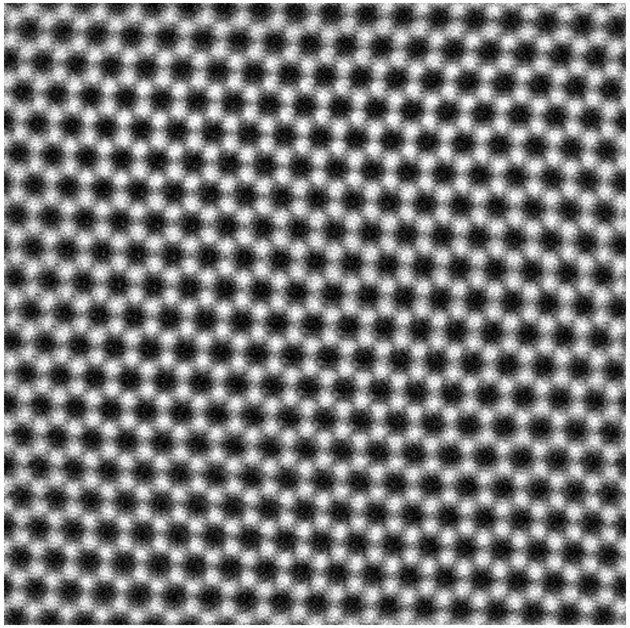
Topological phases with symmetries



SPT = symmetry-protected topological
SET = symmetry-enriched topological

- Interesting symmetries:
  - Charge conservation U(1)
  - Time reversal
  - **Spatial symmetries**

Spatial symmetries



- Crystals have **spatial** symmetries
  - 230 space groups in 3-D
- Free-fermion phases protected by space group symmetries:

  "Topological crystalline insulators" (Liang Fu, 2011)
- **Interacting** topological phases with space group symmetries: ???

Crystalline equivalence principle:

It doesn't matter whether or not the symmetry acts spatially or not – the classification is the same!

Topological phases for internal symmetries

- For internal symmetries (e.g. charge conservation or time reversal), lots of beautiful theory

Bosonic SPTs: Group cohomology (Chen, Gu, Liu, Wen, 2011)

Fermionic SPTs: Group supercohomology (Gu and Wen, 2012)

SETs: G-crossed braided tensor categories

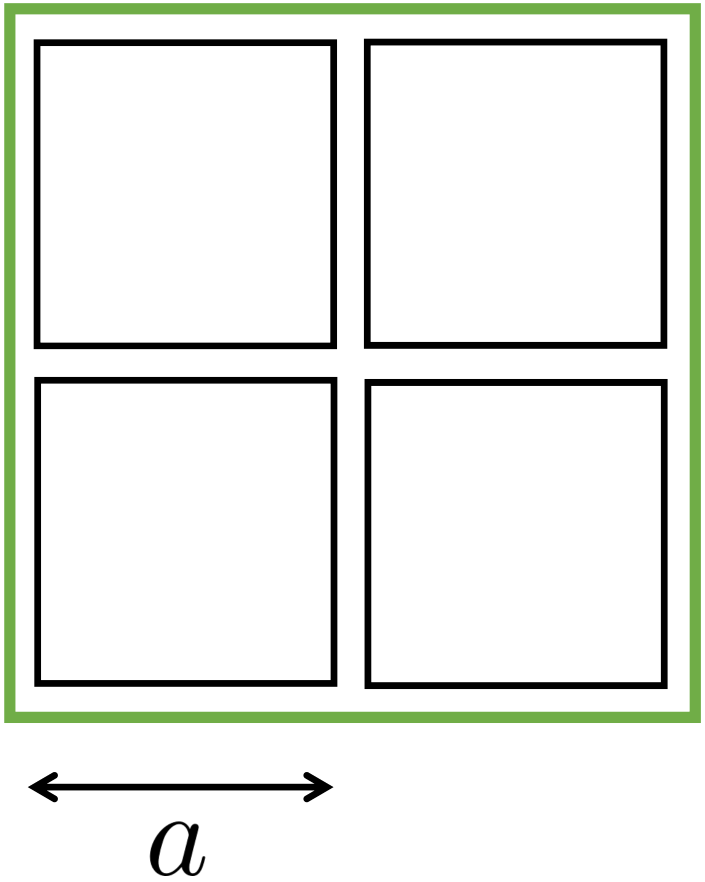
(Etingof et al, 2010; Barkeshli et al, 2014)

Topological quantum field theory

II. The general formalism

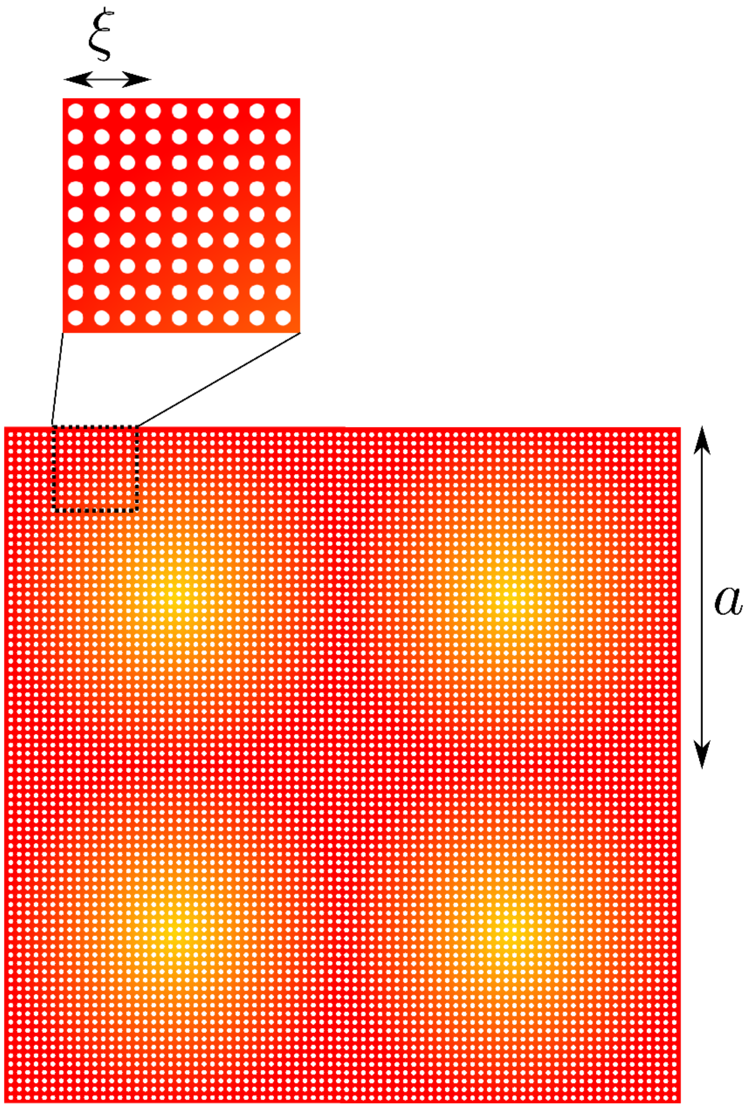
Topological IR limits

Lattice model

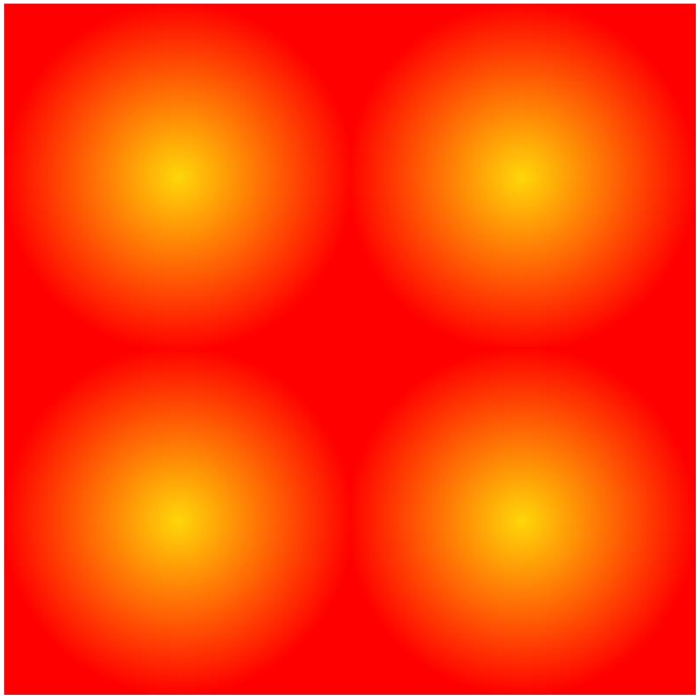


TQFT

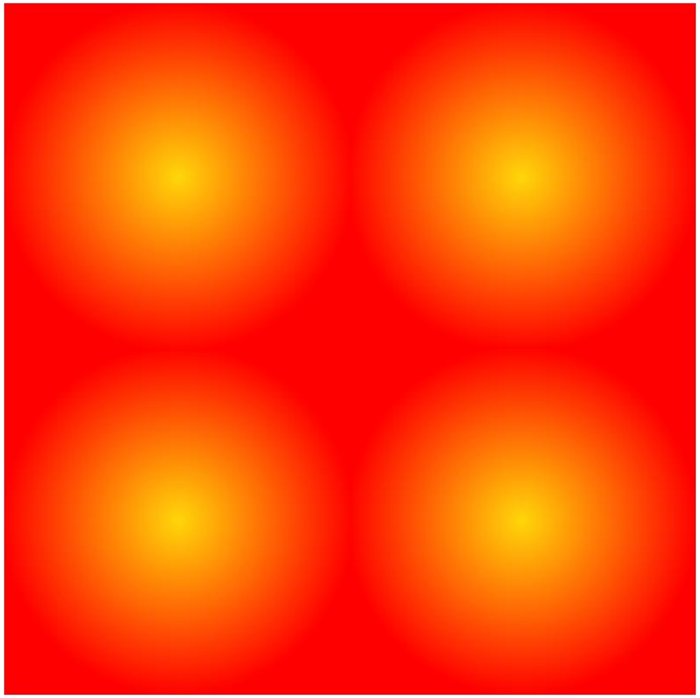
Smooth states



Smooth state



Spatially dependent TQFT



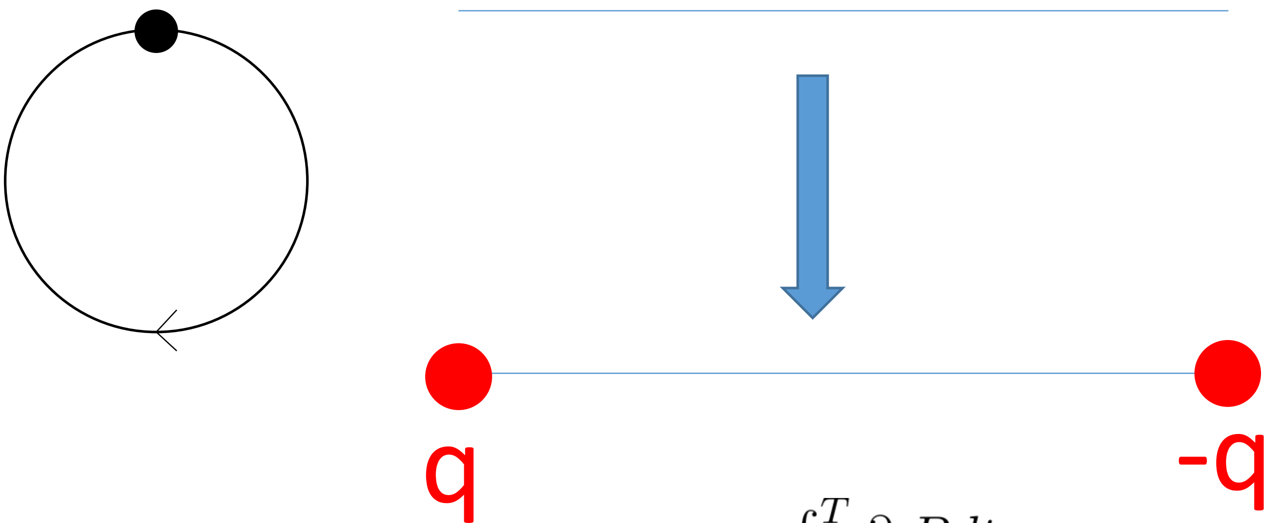
- A spatially dependent TQFT over a space *X* is a continuous map

$$\sigma : X \to \{\text{space of TQFTs}\}$$

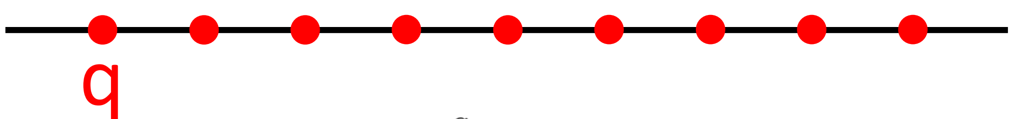
A TQFT is a monoidal functor $Bord_n \to \mathcal{C}$ for some fixed target category $\mathcal{C}$

The space of TQFTs is the geometric realization of the core of $\mathcal{C}$

Spatially dependent TQFT



$$q = \int_0^T \partial_t P dt$$



$$q = \int_0^a \partial_x P$$

- A spatially dependent TQFT over a space *X* is a continuous map

$$\sigma : X \to \{\text{space of TQFTs}\}$$

Theorem

Crystalline equivalence principle:

It doesn't matter whether or not the symmetry acts spatially or not – the classification is the same!

(If $X = \mathbb{R}^d$)

III. Some physical pictures

Equivariant cohomology

Classification of bosonic SPT phases

(Internal symmetries)

$$\text{classif.} \approx \mathcal{H}^{d+1}(G, U(1))$$

$$X = \mathbb{R}^d$$

(Spatial symmetries)

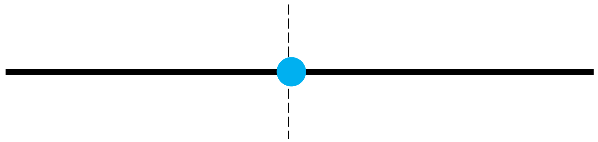
$$\text{classif.} \approx \mathcal{H}_G^{d+1}(X, U(1))$$

Spectral sequence

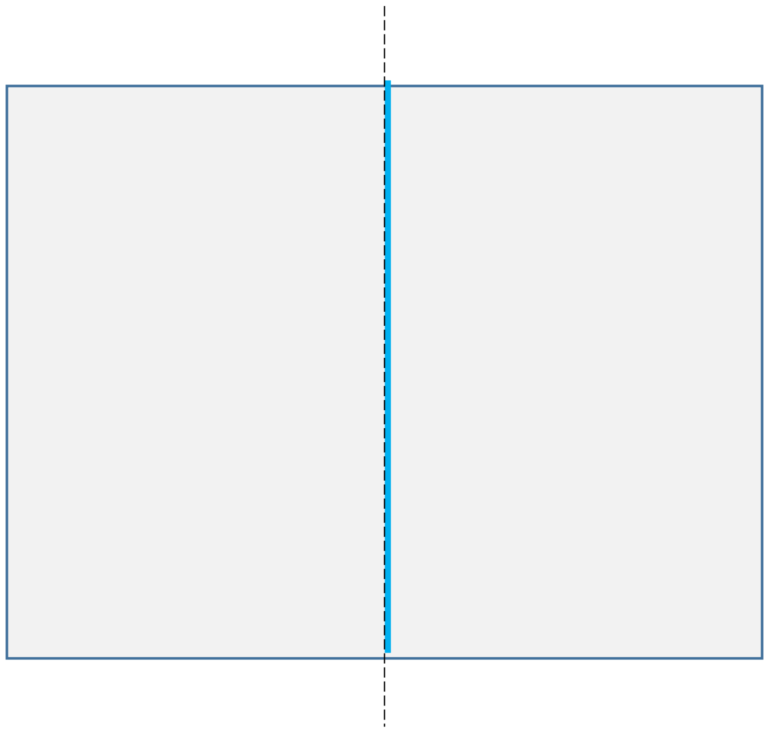
[Song, Huang, Fu, Hermele, PRX ‘16]
[Huang, Song, Huang, Hermele, arXiv:1705.09243]

Examples of equivariant cohomology

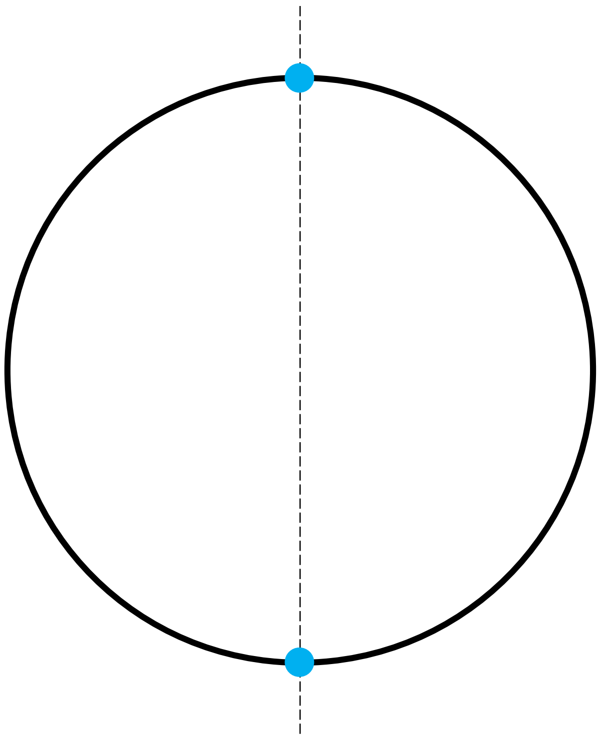
[Song, Huang, Fu, Hermele, PRX ‘16]
[Huang, Song, Huang, Hermele, arXiv:1705.09243]



Classification: $\mathbb{Z}_2$



Classification: $H^2(G_{\text{int}} \times \mathbb{Z}_2, U(1))$



Classification:
$\mathbb{Z}_2 \times \mathbb{Z}_2$

Examples of equivariant cohomology

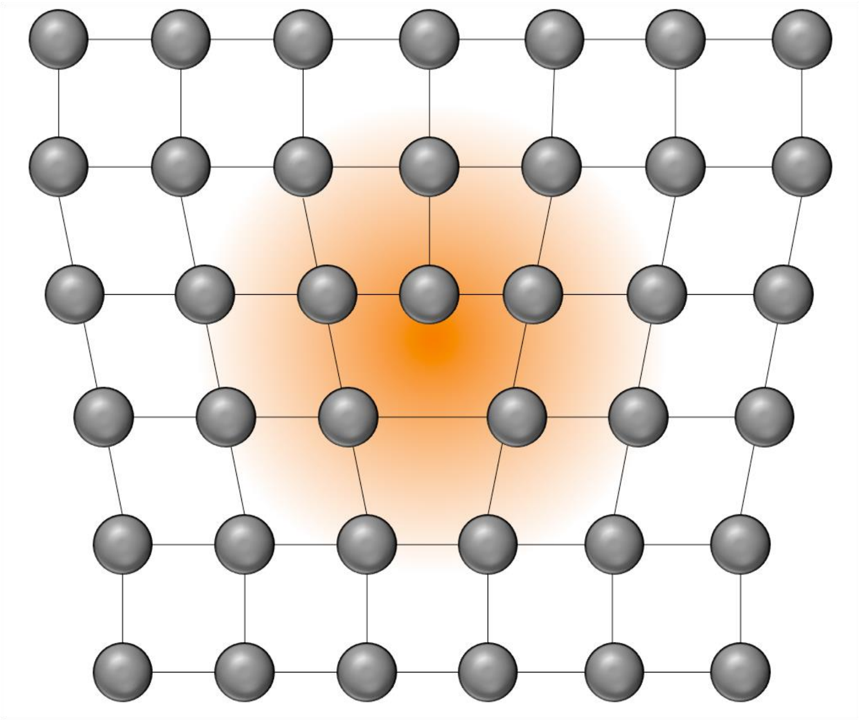
Gauging spatial symmetries

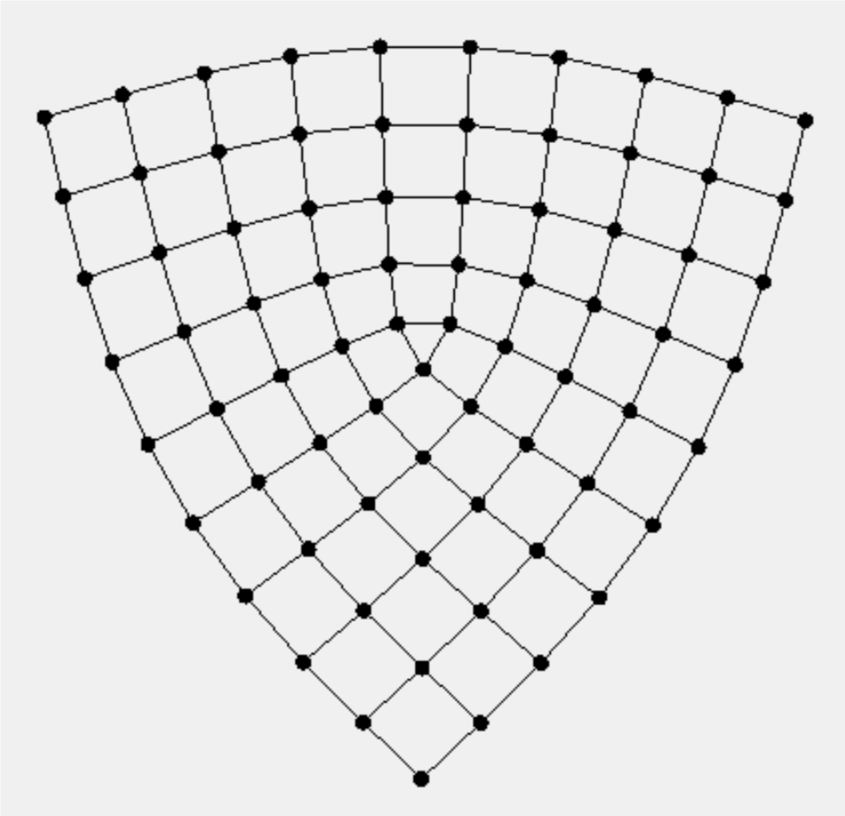
- A spatially dependent TQFT over a space *X* is a continuous map

$$\sigma : X \to \{\text{space of TQFTs}\}$$

- A spatially dependent TQFT **with spatial symmetry G** is **equivalent** a TQFT with a background "crystalline gauge field" (TQFTs can be "gauged").

Gauge field for spatial symmetry





Dislocation
= gauge flux for translational symmetry

Disclination
= gauge flux for rotational symmetry

Discrete gravitational response??

The 230-fold way

Number	Name	Classification
1	P1	0
2	$P\bar{1}$	$\mathbb{Z}_2^{\times 8}$
3	P2	$\mathbb{Z}_2^{\times 4}$
4	$P2_1$	0
5	C2	$\mathbb{Z}_2^{\times 2}$
6	Pm	$\mathbb{Z}_2^{\times 4}$
7	Pc	0
8	Cm	$\mathbb{Z}_2^{\times 2}$
9	Cc	0
10	P2/m	$\mathbb{Z}_2^{\times 18}$
11	$P2_1/m$	$\mathbb{Z}_2^{\times 6}$
12	C2/m	$\mathbb{Z}_2^{\times 11}$
13	P2/c	$\mathbb{Z}_2^{\times 6}$
14	$P2_1/c$	$\mathbb{Z}_2^{\times 4}$
15	C2/c	$\mathbb{Z}_2^{\times 5}$
16	P222	$\mathbb{Z}_2^{\times 16}$
17	$P222_1$	$\mathbb{Z}_2^{\times 4}$
18	$P2_12_12$	$\mathbb{Z}_2^{\times 2}$
19	$P2_12_12_1$	0
20	$C222_1$	$\mathbb{Z}_2^{\times 2}$
21	C222	$\mathbb{Z}_2^{\times 9}$
22	F222	$\mathbb{Z}_2^{\times 8}$
23	I222	$\mathbb{Z}_2^{\times 8}$
24	$I2_12_12_1$	$\mathbb{Z}_2^{\times 3}$
25	Pmm2	$\mathbb{Z}_2^{\times 16}$
26	$Pmc2_1$	$\mathbb{Z}_2^{\times 4}$
27	Pcc2	$\mathbb{Z}_2^{\times 4}$
28	Pma2	$\mathbb{Z}_2^{\times 4}$
29	$Pca2_1$	0
30	Pnc2	$\mathbb{Z}_2^{\times 2}$
31	$Pmn2_1$	$\mathbb{Z}_2^{\times 2}$
32	Pba2	$\mathbb{Z}_2^{\times 2}$
33	$Pna2_1$	0

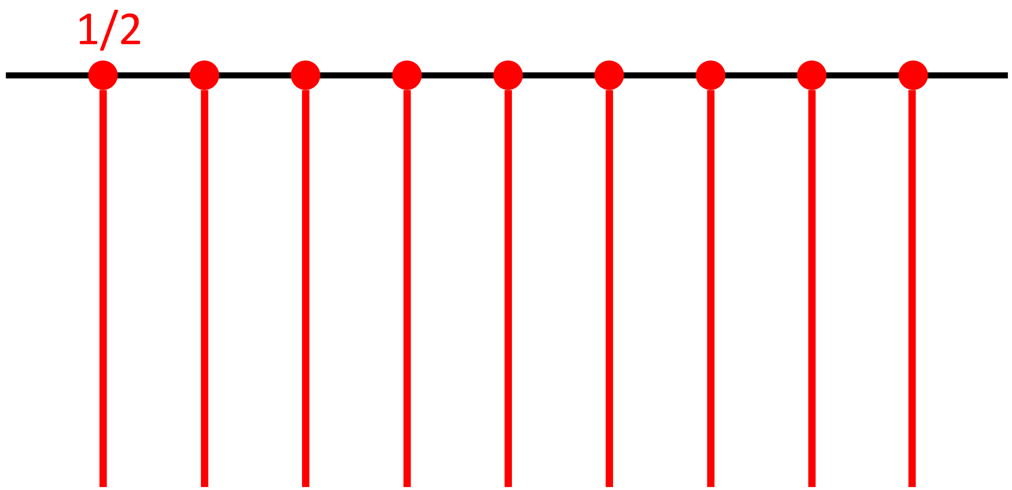
Number	Name	Classification
40	Ama2	$\mathbb{Z}_2^{\times 3}$
41	Aea2	$\mathbb{Z}_2$
42	Fmm2	$\mathbb{Z}_2^{\times 6}$
43	Fdd2	$\mathbb{Z}_2$
44	Imm2	$\mathbb{Z}_2^{\times 8}$
45	Iba2	$\mathbb{Z}_2^{\times 2}$
46	Ima2	$\mathbb{Z}_2^{\times 3}$
47	Pmmm	$\mathbb{Z}_2^{\times 42}$
48	Pnnn	$\mathbb{Z}_2^{\times 10}$
49	Pccm	$\mathbb{Z}_2^{\times 17}$
50	Pban	$\mathbb{Z}_2^{\times 10}$
51	Pmma	$\mathbb{Z}_2^{\times 17}$
52	Pnna	$\mathbb{Z}_2^{\times 4}$
53	Pmna	$\mathbb{Z}_2^{\times 10}$
54	Pcca	$\mathbb{Z}_2^{\times 5}$
55	Pbam	$\mathbb{Z}_2^{\times 10}$
56	Pccn	$\mathbb{Z}_2^{\times 4}$
57	Pbcm	$\mathbb{Z}_2^{\times 5}$
58	Pnnm	$\mathbb{Z}_2^{\times 9}$
59	Pmmn	$\mathbb{Z}_2^{\times 10}$
60	Pbcn	$\mathbb{Z}_2^{\times 3}$
61	Pbca	$\mathbb{Z}_2^{\times 2}$
62	Pnma	$\mathbb{Z}_2^{\times 4}$
63	Cmcm	$\mathbb{Z}_2^{\times 10}$
64	Cmce	$\mathbb{Z}_2^{\times 7}$
65	Cmmm	$\mathbb{Z}_2^{\times 26}$
66	Cccm	$\mathbb{Z}_2^{\times 13}$
67	Cmme	$\mathbb{Z}_2^{\times 17}$
68	Ccce	$\mathbb{Z}_2^{\times 7}$
69	Fmmm	$\mathbb{Z}_2^{\times 20}$
70	Fddd	$\mathbb{Z}_2^{\times 6}$
71	Immm	$\mathbb{Z}_2^{\times 22}$
72	Ibam	$\mathbb{Z}_2^{\times 10}$

Number	Name	Classification
79	I4	$\mathbb{Z}_2 \times \mathbb{Z}_4$
80	$I4_1$	$\mathbb{Z}_2$
81	$P\bar{4}$	$\mathbb{Z}_2^{\times 3} \times \mathbb{Z}_4^{\times 2}$
82	$I\bar{4}$	$\mathbb{Z}_2^{\times 2} \times \mathbb{Z}_4^{\times 2}$
83	P4/m	$\mathbb{Z}_2^{\times 12} \times \mathbb{Z}_4^{\times 2}$
84	$P4_2/m$	$\mathbb{Z}_2^{\times 11}$
85	P4/n	$\mathbb{Z}_2^{\times 3} \times \mathbb{Z}_4^{\times 2}$
86	$P4_2/n$	$\mathbb{Z}_2^{\times 4} \times \mathbb{Z}_4$
87	I4/m	$\mathbb{Z}_2^{\times 8} \times \mathbb{Z}_4$
88	$I4_1/a$	$\mathbb{Z}_2^{\times 3} \times \mathbb{Z}_4$
89	P422	$\mathbb{Z}_2^{\times 12}$
90	$P42_12$	$\mathbb{Z}_2^{\times 4} \times \mathbb{Z}_4$
91	$P4_122$	$\mathbb{Z}_2^{\times 3}$
92	$P4_12_12$	$\mathbb{Z}_2$
93	$P4_222$	$\mathbb{Z}_2^{\times 12}$
94	$P4_22_12$	$\mathbb{Z}_2^{\times 5}$
95	$P4_322$	$\mathbb{Z}_2^{\times 3}$
96	$P4_32_12$	$\mathbb{Z}_2$
97	I422	$\mathbb{Z}_2^{\times 8}$
98	$I4_122$	$\mathbb{Z}_2^{\times 5}$
99	P4mm	$\mathbb{Z}_2^{\times 12}$
100	P4bm	$\mathbb{Z}_2^{\times 4} \times \mathbb{Z}_4$
101	$P4_2cm$	$\mathbb{Z}_2^{\times 6}$
102	$P4_2nm$	$\mathbb{Z}_2^{\times 5}$
103	P4cc	$\mathbb{Z}_2^{\times 3}$
104	P4nc	$\mathbb{Z}_2 \times \mathbb{Z}_4$
105	$P4_2mc$	$\mathbb{Z}_2^{\times 9}$
106	$P4_2bc$	$\mathbb{Z}_2^{\times 2}$
107	I4mm	$\mathbb{Z}_2^{\times 7}$
108	I4cm	$\mathbb{Z}_2^{\times 4}$
109	$I4_1md$	$\mathbb{Z}_2^{\times 4}$
110	$I4_1cd$	$\mathbb{Z}_2$
111	$P\bar{4}2m$	$\mathbb{Z}_2^{\times 13}$

93 | $P4_222$ | $\mathbb{Z}_2^{\times 12}$

Lieb-Schultz-Mattis theorem

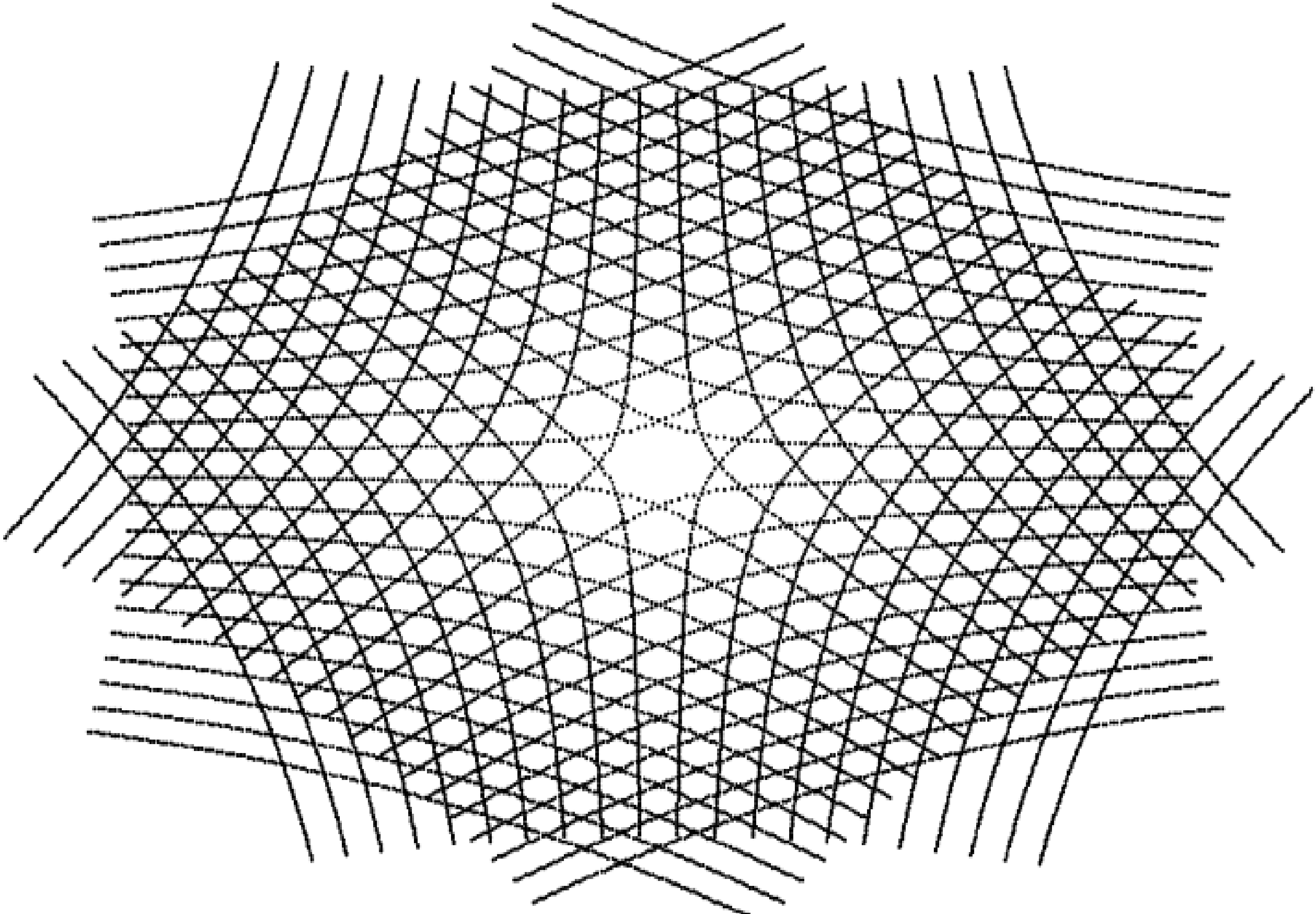
$$G = SO(3) \times \mathbb{Z}$$



Spectral sequence

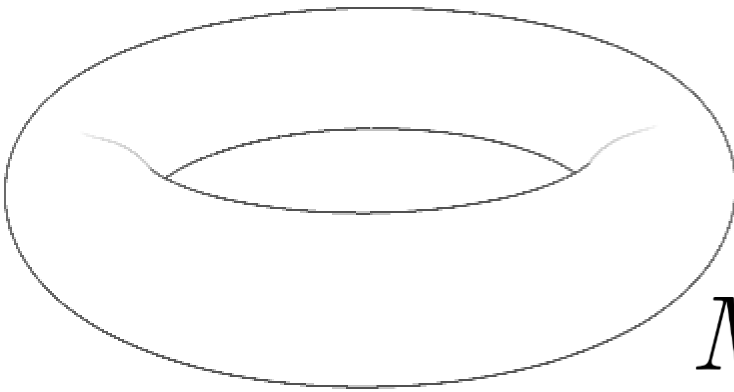
$$\mathcal{H}_G^{d+2}(X, U(1))$$

Thank you!



Gauge fields for spatial symmetry

$$X = \mathbb{R}^d$$



M

$$f(x_+) = g f(x_-)$$

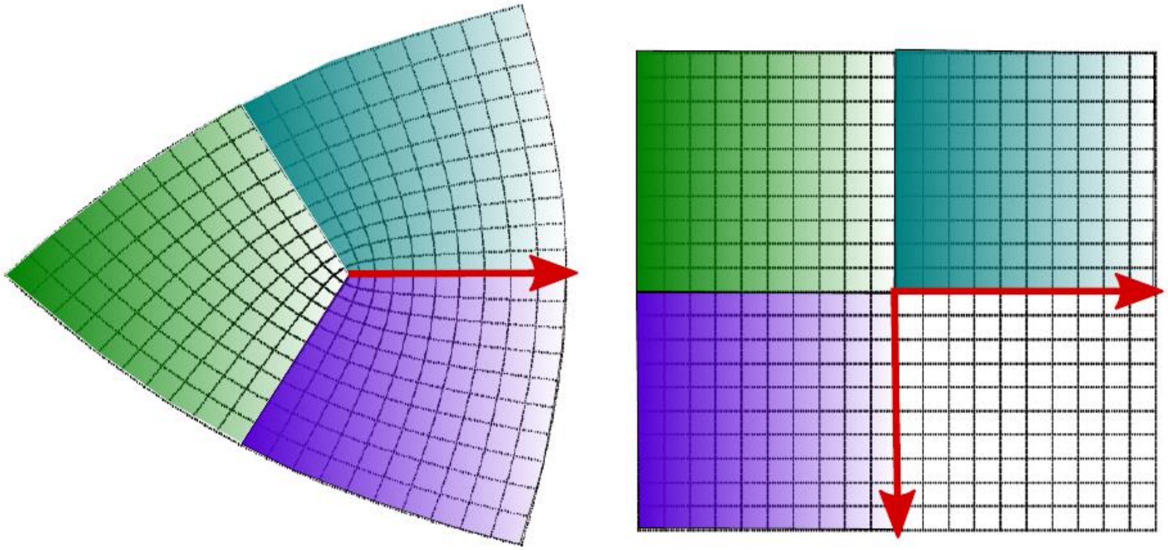
$$M \xrightarrow{f} X$$

$$\begin{array}{ccc} P & \xrightarrow{\pi} & M \\ \downarrow{\scriptstyle \hat{f}} & & \\ X & & \end{array}$$

π is a principal G-bundle

$\hat{f}$ is G-equivariant

Example of crystalline gauge field



$M = \mathbb{R}^2 \setminus \{0\}$ $X = \mathbb{R}^2$

$f : M \to X$ $f(z) = z^{3/4}$